

A short comparison between m_{T2} and m_{CT}

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ABSTRACT: We compare m_{T2} with m_{CT} ; both are kinematic variables designed to find relationships between masses of pair-produced new states with symmetric decay chains. We find that for massless visible particles m_{CT} equals m_{T2} in a particular limit. We identify advantages and disadvantages to the use of each variable. Tovey's paper on m_{CT} also introduced a powerful concept of extracting mass information from an analysis at intermediate stages of a symmetric decay chain. We suggest that m_{T2} is a better tool for performing this analysis than m_{CT} due to m_{T2} 's better properties under initial state radiation.

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Dark matter's likely signature in a hadron collider will be missing transverse momentum. The stability of dark matter suggests a charge or conservation law that requires dark matter particles be produced in pairs at colliders. The kinematic variables m_{T2} introduced by Lester and Summers [1] and m_{CT} introduced by Tovey [2] aid in the task of determining the mass of new states that decay to dark matter particles at hadron colliders. Although m_{T2} has been used extensively (see [3, 4, 5, 6, 7] for a few examples), the variable m_{CT} is new but shares many similarities and differences with m_{T2} . This note briefly defines m_{T2} and m_{CT} , explains when they give identical results, when they differ, and comments on benefits of each in their intended applications.

Both variables assume a pair-produced new-particle state followed by each branch decaying symmetrically to visible states and dark-matter candidates which escape detection and appear as missing transverse momentum. Fig 1 is the simplest example on-which we can meaningfully compare the two kinematic quantities. The figure shows two partons colliding and producing some observed initial state radiation (ISR) with four momenta g and an on-shell, pair-produced new state Y . On each branch, Y decays to on-shell states X and v_1 with masses m_X and m_{v_1} , and X then decays to on-shell states N and v_2 with masses m_N and m_{v_2} . The four-momenta of v_1 , v_2 and N are respectively α_1 , α_2 and p on one branch and β_1 , β_2 and q in the other branch. The missing transverse momenta \vec{P}_T is given by the transverse part of $p + q$.

First we describe m_{T2} . The variable m_{T2} accepts three inputs: χ (an assumed mass of the two particles carrying away missing transverse momenta), α and β (the visible momenta of each branch), and $\vec{P}_T = (p + q)_T$ (the missing transverse momenta). The variable m_{T2} is the minimum mass of the pair of parent particles compatible with the observed particles' four momenta and an assumed mass for particles carrying away the missing momenta. We can define m_{T2} in terms of the transverse mass of each branch where we minimize the maximum of the two transverse masses over the unknown split between p and q of the

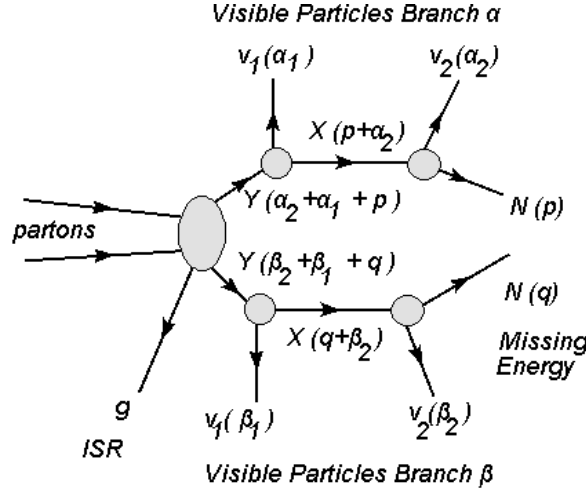


Figure 1: A simplest topology with which we compare m_{T2} and m_{CT} .

overall missing transverse momenta:

$$m_{T2}^2(\chi_N, \alpha, \beta, \vec{p}_T) = \min_{p_T + q_T = \vec{p}_T} [\max \{m_T^2(\alpha, p), m_T^2(\beta, q)\}]. \quad (1)$$

In this expression χ_N is the assumed mass of N , α and β are the four momenta of the visible particles in the two branches, the transverse mass is given by $m_T^2(\alpha, p) = m_\alpha^2 + \chi_N^2 + 2(E_T(p)E_T(\alpha) - p_T \cdot \alpha_T)$ and the transverse energy $E_T(p) = \sqrt{p_T^2 + \chi_N^2}$ is determined from the transverse momentum of p and the assumed mass of the particle associated with momentum p . An analytic formula for the case with no transverse ISR can be found in the appendix of [4]. For each event, the quantity $m_{T2}(\chi_N = m_N, \alpha_1 + \alpha_2, \beta_1 + \beta_2, \vec{p}_T)$ gives the smallest mass for the parent particle compatible with the event's kinematics. Under ideal assumptions, the mass of the parent particle Y is given by the end-point of the distribution of this m_{T2} parameter over a large number of events like fig 1. Because a priori we do not know m_N , we need some other mechanism to determine m_N ¹. We use χ to distinguish assumed values of the masses (χ_Y, χ_X, χ_N) from the true values for the masses (m_Y, m_X, m_N). Because of this dependence on the unknown mass, we should think of $\max m_{T2}$ as providing a relationship or constraint between the mass of Y and the mass of N . This forms a surface in the (χ_Y, χ_X, χ_N) space on which the true mass will lie. We express this relationship as $\chi_Y(\chi_N)$ ².

Tovey [2] recently defined a new variable m_{CT} which has many similarities to m_{T2} . The variable is defined as

$$m_{CT}^2(\alpha_1, \beta_1) = (E_T(\alpha_1) + E_T(\beta_1))^2 - (\alpha_{1T} - \beta_{1T})^2. \quad (2)$$

Tovey's goal is to identify another constraint between masses in the decay chain. He observes that in the rest frame of Y the momentum of the back-to-back decay products X and v_1 is given by

$$(k_*(m_Y, m_X, m_{v_1}))^2 = \frac{(m_Y^2 - (m_{v_1} + m_X)^2)(m_Y^2 - (m_{v_1} - m_X)^2)}{4m_Y^2} \quad (3)$$

where k_* is the two-body excess momentum parameter (2BEMP)³. In the absence of transverse ISR ($g_T = 0$) and if the visible particles are effectively massless ($m_{v_1} = 0$), Tovey observes that $\max m_{CT}(\alpha_1, \beta_1)$ is given by $2k_*$; this provides an equation of constraint between m_Y and m_X . Tovey observes that if we could do this analysis at various stages along the symmetric decay chain all the masses could be determined.

The big advantage of m_{CT} is in its computational simplicity. Also, m_{CT} is intended to only be calculated once per event instead of at a variety of choices of χ . In contrast, m_{T2}

¹The true m_N and m_Y can be found in the case where Y undergoes a three-body (X is off-shell) through kinks in m_{T2} [5, 6, 8] or when combined with endpoints from other distribution (like $\max m_{ll}$) [7].

²In principle this surface would be considered a function of $\chi_Y(\chi_X, \chi_N)$, but m_{T2} makes no reference to the mass of X and the resulting constraints are therefore independent of any assumed value of the mass of X .

³Tovey refers to this as the 2-body mass parameter \mathcal{M}_i . We feel calling this a mass is a bit misleading so we are suggesting 2BEMP.

is a more computationally intensive parameter to compute; but this is aided by the use of a shared repository of community tested C++ libraries found at [9].

How are these two variables similar? Both m_{CT} and m_{T2} , in the absence of ISR, are invariant under back-to-back boosts of the parent particles momenta [6]. The variable m_{CT} equals $m_{T2}(\chi = 0)$ in the special case where $\chi = 0$ and when the visible particles are massless ($\alpha_1^2 = \beta_1^2 = 0$) and there is no transverse ISR ($g_T = 0$)

$$m_{CT}(\alpha_1, \beta_1) = m_{T2}(\chi = 0, \alpha_1, \beta_1, \not{p}_T = (p + q + \alpha_2 + \beta_2)_T) \quad \text{if } \alpha_1^2 = \beta_1^2 = 0. \quad (4)$$

$$= 2(\alpha_{1T} \cdot \beta_{1T} + |\alpha_{1T}| |\beta_{1T}|). \quad (5)$$

The m_{CT} side of the equation is straight forward. The m_{T2} side of the expression can be derived analytically using the formula for m_{T2} given in [4]; we also outline a short proof in the appendix. Eq(4) uses a m_{T2} in a unconventional way; we group the observed momenta of the second decay products into the missing transverse momenta. In this limit, both share an endpoint of $2k_* = (m_Y^2 - m_X^2)/m_Y$. To the best of our knowledge, this endpoint was first pointed out by Cho *et.al.* [5]⁴. We find it surprising that a physical relationship between the masses follows from m_{T2} evaluated at a non physical χ . In the presence of ISR, eq(4) is no longer an equality. Furthermore in the presence of the ISR, the end point of the distribution given by either side of eq(4) exceeds $2k_*$. In both cases, we will need to solve a combinatoric problem of matching visible particles to their decay order and branch of the event which is beyond the scope of this paper.

In the case where the visible particle v_1 is massive, the two parameters give different end-points

$$\max m_{CT}(\alpha_1, \beta_1) = \frac{m_Y^2 - m_X^2}{m_Y} + \frac{m_{v_1}^2}{m_Y} \quad (6)$$

$$\max m_{T2}(\chi = 0, \alpha_1, \beta_1, \not{p}_T = (p + q + \alpha_2 + \beta_2)_T) = \sqrt{m_{v_1}^2 + 2(k_*^2 + k_* \sqrt{k_*^2 + m_{v_1}^2})} \quad (7)$$

where k_* is given by eq(3). Unfortunately, there is no new information about the masses in these two endpoints. If we solve eq(6) for m_X and substitute this into eq(7) and (3), all dependence on m_Y is eliminated.

Tovey's idea of analyzing the different steps in a symmetric decay chain to extract the masses is powerful. Up until now, we have been analyzing both variables in terms of the first decay products of Y . This restriction is because m_{CT} requires no transverse ISR to give a meaningful endpoint. If we were to try and use α_2 and β_2 to find a relationship between m_X and m_N , then we would need to consider the transverse ISR to be $(g + \alpha_1 + \beta_1)_T$ which is unlikely to be zero.

We suggest m_{T2} is a better variable with which to implement Tovey's idea of analyzing the different steps in a symmetric decay chain because of its ISR properties. With and without ISR, m_{T2} 's endpoint gives the correct mass of the parent particle when we assume

⁴The endpoint given by Cho *et.al.* is violated for non-zero ISR at $\chi_N < m_N$ and $\chi_N > m_N$.

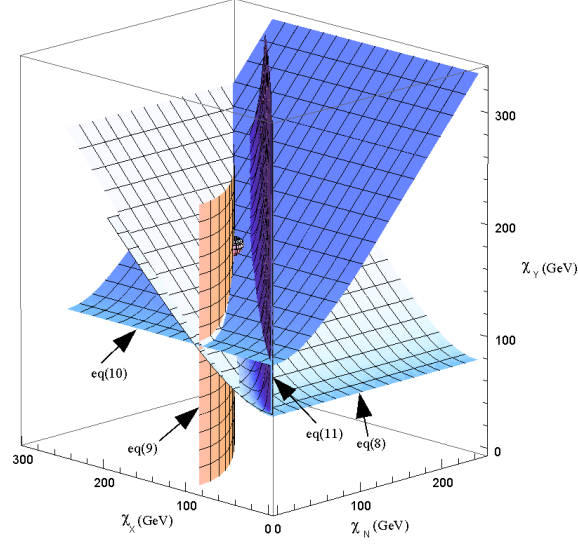


Figure 2: Shows constraints from $\max m_{T2}$ used with different combinations as described in eqs(8,9,10) and the $\max m_{12}$ described in eq(12). Intersection is at the true mass (97 GeV, 144 GeV, 181 GeV) shown by sphere. Events include ISR but otherwise ideal conditions: no background, resolution, or combinatoric error.

the correct value of the missing-energy-particle's mass ⁵. For this reason, $\max m_{T2}$ gives a meaningful relationship between masses (m_Y, m_X, m_N) for all three symmetric pairings of the visible particles across the two branches. A relationship between m_Y and m_X is given by

$$\chi_Y(\chi_X) = \max m_{T2}(\chi_X, \alpha_1, \beta_1, \not{P}_T = (p + q + \alpha_2 + \beta_2)_T). \quad (8)$$

A relationship between m_X and m_N can be found by computing

$$\chi_X(\chi_N) = \max m_{T2}(\chi_N, \alpha_2, \beta_2, \not{P}_T = (p + q)_T) \quad (9)$$

where we have grouped $\alpha_1 + \beta_1$ with the g as a part of the ISR. A relationship between m_Y and m_N can be found by using m_{T2} in the traditional manner giving

$$\chi_Y(\chi_N) = \max m_{T2}(\chi_N, \alpha_1 + \alpha_2, \beta_1 + \beta_2, \not{P}_T = (p + q)_T). \quad (10)$$

Lastly, we can form a distribution from the invariant mass of the visible particles on each branch $m_{12}^2 = (\alpha_1 + \alpha_2)^2$ or $m_{12}^2 = (\beta_1 + \beta_2)^2$. The endpoint of this distribution gives a relationship between m_Y , m_X , and m_N given by

$$\max m_{12}^2 = \frac{(m_Y^2 - m_X^2)(m_X^2 - m_N^2)}{m_X^2}. \quad (11)$$

⁵In principle we could plot the $\max m_{T2}(\chi_X, \alpha_1, \beta_1, \not{P}_T = (\alpha_2 + \beta_2 + p + q)_T)$ vs χ_X as a function of transverse ISR and the value of χ_X at which the end point is constant would give the correct value of m_X ; at which point the distributions end point would give the correct m_Y . In practice we probably will not have enough statistics of ISR events.

Solving this expression for m_Y gives the relationship

$$\chi_Y^2(\chi_N, \chi_X) = \frac{\chi_X^2((\max m_{12}^2) + \chi_X^2 - \chi_N^2)}{\chi_X^2 - \chi_N^2}. \quad (12)$$

Fig 2 shows the constraints from eqs(8,9,10,12) in an ideal simulation using ($m_Y = 181$ GeV, $m_X = 144$ GeV, $m_N = 97$ GeV), 1000 events, and massless visible particles, and ISR added with an exponential distribution with a mean of 50 GeV. These four surfaces in principle intersect at a single point (m_Y, m_X, m_N) given by the sphere in the figure 2. Unfortunately, all these surfaces intersect the correct masses at a shallow angles so we have a sizable uncertainty along the direction of the sum of the masses and a tight constraints in the perpendicular directions. In other words, the mass differences are well-determined but not the mass scale. From here one could use a shape fitting technique like that described in [7] to find a constraint on the sum of the masses. Tovey's suggestion for extracting information from these intermediate stages of a symmetric cascade chain clearly provides more constraints to isolate the true mass than one would find from only using the one constraint of eq(10) as described in [5]. However, Tovey's suggestion is more feasible using the m_{T2} rather than m_{CT} because the constraint surfaces derived from m_{T2} intersect the true masses even with ISR.

In summary, we have compared and contrasted m_{CT} with m_{T2} . The variable m_{CT} is a special case of m_{T2} given by eq(4) when ISR can be neglected and when the visible particles are massless. In this case, the end-point of this distribution gives $2k_*$, twice the two-body excess momentum parameter (2BEMP). If $m_{v1} \neq 0$, the two distributions have different endpoints but no new information about the masses. In the presence of ISR the two functions are not equal; both have endpoints that exceed $2k_*$. Because of it's better properties in the presence of ISR, m_{T2} is a better variable for the task of extracting information from each step in the decay chain. Extracting this information requires solving combinatoric problems which are beyond the scope of this paper.

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Appendix: Verifying m_{T2} in eq(5)

We derived the m_{T2} side of eq(5) by following the analytic solution given by Barr and Lester in [4]. In this appendix, we outline how to verify that m_{T2} is indeed given by

$$m_{T2}(\chi = 0, \alpha, \beta, \not{P}_T = -\alpha_T - \beta_T) = 2(\alpha_T \cdot \beta_T + |\alpha_T| |\beta_T|) \quad (13)$$

when $\alpha^2 = \beta^2 = 0$ and $p^2 = q^2 = \chi^2 = 0$ and $g_T = 0$. To do this we note that m_{T2} can also be defined as the minimum value of $(\alpha + p)^2$ minimized over p and q subject to the conditions $p^2 = q^2 = \chi^2$ (on-shell missing energy state), and $(\alpha + p)^2 = (\beta + q)^2$ (equal on-shell parent-particle state), and $(\alpha + \beta + p + q + g)_T = 0$ (conservation of transverse momentum) [7].

The solution which gives eq(13) has $p_T = -\beta_T$ and $q_T = -\alpha_T$ with the rapidity of $p(q)$ equal to the rapidity of α (β). We now verify that this solution satisfies all the constraints listed above. Transverse momentum conservation is satisfied trivially: $(\alpha + \beta + p + q)_T = (\alpha + \beta - \alpha - \beta)_T = 0$. The constraint to have the parent particles on-shell can be verified with $2|\alpha_T||p_T| - 2\vec{p}_T \cdot \vec{\alpha}_T = 2|\beta_T||q_T| - 2\vec{q}_T \cdot \vec{\beta}_T = 2|\beta_T||\beta_T| + 2\vec{\alpha}_T \cdot \vec{\beta}_T$.

Now all that remains is to show that the parent particle's mass is a minimum with respect to ways in which one splits up the missing transverse momentum between p_T and q_T while satisfying the above constraints. We take p and q to be a small deviation from the stated solution $p_T = -\beta_T + \delta_T$ and $q_T = -\alpha_T - \delta_T$ where δ_T is the small deviation in the transverse plane. We keep p and q on shell at $\chi = 0$. The terms p_o , p_z , q_o , q_z are maintained at their minimum by keeping the rapidity of p and q equal to α and β . The condition that the parent particles are on-shell and equal is satisfied for a curve of values for δ_T . The deviation tangent to this curve near $|\delta_T| = 0$ is given by $\delta_T(\lambda) = \lambda \hat{z} \times (\alpha_T|\beta_T| + \beta_T|\alpha_T|)$ where \times is a cross product, \hat{z} denotes the beam direction, and we parameterized the magnitude by the scalar λ . Finally, we can substitute p and q with the deviation $\delta_T(\lambda)$ back into the expression for the parent particle's mass $(\alpha + p)^2$ and verify that $2(\alpha_T \cdot \beta_T + |\alpha_T||\beta_T|)$ at $\lambda = 0$ is indeed the minimum with respect to changes in λ .

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